Macro Modeling with Chain-Type GDP

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I Introduction

Any empirical investigation would benefit from an improvement in the quality of the underlying data. In macroeconomic modeling and forecasting, increasing the timeliness, and reducing the sample bias and variance of the data upon which these activities depend, will help to improve both our understanding of the structure of the economy and its behavior through time, and, by extension, our ability to forecast it and pursue appropriate macroeconomic policies. Unfortunately, in most cases, improvements in economic data do not come without a cost. Traditionally, such improvements required the collection of more data, of different types, from an increased number of collection points, as well as the application of more advanced processing, and so on, all of which involve a sometimes significant cost. Moreover, governmental statistical agencies are being asked to do more with less. These times of rapid technological and structural change may be raising the importance of high quality economic data for both economic policy-makers and private-sector practitioners, while coincidentally tight governmental budgets are resulting in the resources provided to the statistical agencies being frozen or reduced.

Fortunately, the old axiom "work smarter, not harder" may have some relevance. The collection and processing of economic data is likely to benefit significantly from "working smarter" until such time that additional resources become available which also allow us to capture the benefits of "working harder." Specifically, there are two ways in which working smarter will pay big dividends with tolerable additional costs. First, the re-engineering of economic data, as in the switch from fixed-weight to chain-type GDP, results in a dramatic improvement in the quality of the aggregate data, with, in principle, no additional data being required. Second, the same technological advances which are the greatest motivating factor requiring the re-engineering of our aggregate data, will also, in time, improve the quantity, quality, and timeliness of the raw data, while lowering its collection cost. Private business is rapidly applying electronic data collection technologies to its own data. Once our statistical agencies can tap into these data networks, near real-time economic data drawn from huge samples will not be far behind. The subject of this paper, however, is to address one small, but important, part of the first way to higher quality data, namely the adaptations required in macro modeling and forecasting in switching to chain-type GDP.

Far and away, the primary reason to re-engineer aggregate economic statistics and replace fixed-weight measures with chain-type measures is to eliminate the substitution bias that can afflict fixed-weight indices. This becomes especially important when rapid and sustained changes in relative prices occur, as is now the case with computers and other high-technology items. Macroeconomic analysis and modeling requires the ability to accurately decompose changes in nominal aggregates into the price and real components. Fixed-weight indices, as we shall see below, are simply not well suited to this task, and are easily dominated by chain-type measures. We should offer an even stronger statement on this point, namely, fixed-weight indices can be *significantly misleading* when it comes to the measurement of real GDP and other key aggregates. As a result, despite the considerable upheaval in the user community that a switch to chain-type GDP may cause, it is well worth it.

Fixed-weight measures, as the name implies, employ the weights of a specific time period in the identities that aggregate the detailed components. Statisticians have long recognized the problem of substitution bias, and they also recognized that it becomes more severe the further the distance in time from the period supplying the fixed weights. As a result, it has been customary to periodically "re-weight" the data using a more recent time period as the source of the fixed weights. This reduces the degree of the substitution bias in the "current" data, but increases it in data further back in time. Thus economic history changes each time this periodic updating of the weighting period occurs. In contrast, chain-type measures are not dependent upon the weights of any *specific* period because they depend on the weights of *every* period and, therefore, there is no need to re-weight them periodically. Chain-type indices are, in this sense, absolute, and independent of the base year. Importantly, comparisons across business cycles, and the measurement of important trends are not subject to revision every time the statistical agency judges that it is time to re-weight the index.

Lest we leave the impression that the switch to chain-type GDP in the US was met with ready acceptance, we should point out some of the serious problems it has caused as well. Chain-type GDP, as it is implemented in the US, is an approximation of a Fisher Ideal index. The immediate and most difficult challenge to anyone accustomed to working with the fixed-weight data is, principally, the nonadditivity of the real components. For generations of macroeconomists who learned in the first chapter of their first book on macroeconomics that GNP = C + I + G + X - M, the news that this identity would no longer "ident" came as a rude shock. There are other requirements imposed on users of the data or problems that must be overcome as a result of the switch to chain-type GDP. Some either are a property of all chain-type measures, or arise out of the details of the particular implementation of the Fisher Ideal index in the US. Including the nonadditivity problem just mentioned, these are:

I.1) Nonadditivity: Real components of GDP no longer sum to GDP.

I.2) Re-estimation: All behavioral equations employing the data must be re-estimated.

I.3) Different Weighting Schemes for Deep History and Recent History: The data is published using two different weighting methods depending on whether the data is recent history or deep history.

I.4) PQ -= Y$: The particular weighting scheme employed in the US results in the quarterly data not being true Fisher Ideal indices. As a result, the product of price times real does not equal nominal.

I.5) Differences in Level of Disaggregation: The construction of chain-type aggregates utilizes the lowest level of detail so that a user aggregating two subaggregates, even using the correct aggregation scheme, will never get a result that exactly matches the official figures.

I.6) Annual Averaging Does Not Hold: The particular weighting scheme utilized by BEA has the property that the quarterly data do not aggregate across time, i.e., the average of any GDP component across the four quarters of a year does not necessarily equal the annual value for that component.

All of these properties pose considerable mechanical difficulties to anyone accustomed to frequently manipulating large amounts of both detailed and aggregated macroeconomic time series, and for macro modelers especially. Dealing with these difficulties is not impossible, but it is not easy either. Done right, it requires modelers to correctly complete three major tasks: **1)** re-estimate all the behavioral relationships, **2)** re-structure the model to be able to accommodate the needs of the various users of the model for alternative weighting schemes, while insuring that the simulation properties are independent of the aggregation scheme, **3)** re-structure the aggregating relationships in the model to reflect the new method(s) of calculating all real quantities, prices, and nominals.

Each of the major tasks will be addressed below, in sections III, IV, and V respectively. But first, in Section II we provide some background on the issue of fixed-weight versus chain-type GDP and the specifics of the implementation of Chain-Type GDP in the US. In section VI we offer some concluding remarks.

II) Background: Implementation of Chain-Type GDP in the US

The reason any of this is of interest is the necessity of decomposing growth in nominal magnitudes across time between that part arising from changes in prices and that part arising from changes in quantities. This is clearly not a problem at the level of an elemental component of GDP. The nominal magnitude is known, the price index is known, so the real quantity (denominated in dollars) is just the nominal quantity divided by the price index. However, this becomes an issue as soon as it is necessary to aggregate two or more components with different price indices.

II.1) Measuring Fixed-Weight GDP and Prices

Begin by considering a simple economy with only consumer and investment goods. In time period t, $Q_{C,t}$ is the physical quantity of consumer goods produced, $Q_{I,t}$ is the physical quantity of investment goods produced, while $p_{C,t}$ and $p_{I,t}$ are their respective prices. The level of nominal consumption is $C\$_t = p_{C,t}Q_{C,t}$, the level of nominal investment is $I\$_t = p_{I,t}Q_{I,t}$, and nominal GDP ($Y\$_t$) is computed by summing nominal consumption and investment:

$$(1) \qquad Y\$_t = C\$_t + I\$_t = p_{C,t}Q_{C,t} + p_{I,t}Q_{I,t}$$

Measured in 1987 prices, the level of real consumption in period t is $C87_t = p_{C,87}Q_{C,t}$, the level of real investment is $I87_t = p_{I,87}Q_{I,t}$. We can define price indices, P, equal to 1 in the base year, such that $P_{C,t}=p_{C,t}/p_{C,87}$ and $P_{I,t}= p_{I,t}/p_{I,87}$. The real quantities can be re-written as $C87_t = p_{C,87}Q_{C,t}\ p_{C,t}/p_{C,t} = C\$_t/P_{C,t}$, i.e. as the ratio of the nominal magnitude to the price index.

Thus expression (1) can be re-written as:

$$(1') \qquad Y\$_t = C\$_t + I\$_t = p_{C,t}Q_{C,t} + p_{I,t}Q_{I,t} = P_{C,t}\,C87_t + P_{I,t}\,I87_t$$

This is the more familiar form where physical quantities are replaced with dollar-denominated real magnitudes, and prices are replaced with price indices which have a value of 1 in the base period.

Real GDP ($Y87_t$) is computed by summing real consumption and investment:

$$(2) \qquad Y87_t = C87_t + I87_t = p_{C,87}Q_{C,t} + p_{I,87}Q_{I,t}\ .$$

Finally, the implicit deflator of Gross Domestic Product ($P87_t$) is computed as the ratio of the current value of quantities to their value in 1987 prices:

$$(3) \qquad P87_t = (p_{C,t}Q_{C,t} + p_{I,t}Q_{I,t}) / (p_{C,87}Q_{C,t} + p_{I,87}Q_{I,t})$$

$$= (C\$_t + I\$_t)/ (C87_t + I87_t)$$

This is how the Bureau of Economic Analysis (BEA) in the US constructed fixed-weight real GDP and its companion price level, except that BEA keeps track not just of two quantities and prices, but over one thousand. For annual data, "t" is defined as a calendar year; for quarterly data, "t" is a calendar quarter. Yearly data can be computed either by averaging the individual prices and quantities annually before aggregation or, with the same result, averaging quarterly data for Y87 and P87 annually after aggregation. Given the definitions of real output and the aggregate price level, it is identically true that nominal GDP is the product of real fixed-weight GDP and the implicit deflator:

(4) $$Y\$_t = P87_t \, Y87_t$$

At the time of the last benchmark revisions using fixed-weight data, these constructs were "re-based" from 1982 to 1987, so that the price indices equaled 1 in 1987, and in the computation of real GDP, quantities were valued at the prices that obtained in 1987 (e.g. $I87_t = p_{I,87}Q_{I,t}$), instead of in 1982. That is, the components of real GDP were <u>re-weighted</u> using 1987 prices. With all price weights equal to 1, expression (2) makes clear that additivity holds throughout.

In the fixed-weight approach to aggregating components of real GDP, the additivity property makes it quite natural to think in terms of the *levels* of real GDP and its components. Strictly speaking, however, Y87, C87, and I87 are indices; and, as the Chief Statistician of the BEA is fond of saying, "there is nothing real about real GDP." Nevertheless, this hasn't stopped users of fixed-weight GDP from thinking it is *real*, and thinking of GDP as fundamentally a "level" concept.

To see how the growth rates of real fixed-weight GDP are affected by the periodic re-basing, begin from expression (2) and calculate the ratio of this period's real GDP to last period's real GDP:

(5) $$Y87_t/Y87_{t-1} = (p_{C,87}Q_{C,t} + p_{I,87}Q_{I,t})/(p_{C,87}Q_{C,t-1} + p_{I,87}Q_{I,t-1})$$

With some manipulation, (5) can be re-written as:

(6) $$Y87_t/Y87_{t-1} = \beta_{C,t-1}\,(Q_{C,t} / Q_{C,t-1}) + \beta_{I,t-1}\,(Q_{I,t} / Q_{I,t-1}),$$

where the weights are defined as the lagged share of real consumption in real GDP, $\beta_{C,t-1} = C87_{t-1}/Y87_{t-1}$, and the lagged share of real investment in real GDP, $\beta_{I,t-1} = I87_{t-1}/Y87_{t-1}$. Since $C\$ = P_C C87$, the real share can be written as $\beta_{C,t-1}=(C\$_{t-1}/Y\$_{t-1})/(P87_{t-1}/P_{C,t-1})$.

There are three things to note about expression (6). First, the period-to-period change in the physical quantities is independent of the base year. Second, in the weights, $\beta_{C,t-1}$ and $\beta_{I,t-1}$, the nominal component share in GDP is independent of the base year. Third, the ratios of $P87_{t-1}/P_{C,t-1}$ and $P87_{t-1}/P_{I,t-1}$, do depend on the base year. For example, in the base period, the ratio is one, but in any other period the ratio could differ from one as a result of relative price changes. Simply moving the base period, changes $P??_{t-1}/P_{C,t-1}$ at every point in time. As a result, the weights , $\beta_{C,t-1}$ and $\beta_{I,t-1}$, also depend on the base period. This is why the periodic re-basing of fixed-weight GDP results in sometimes dramatic revisions in measured growth rates of GDP.

Note also that the weight of the *i*th component depends inversely on the relative price of the *i*th component. As the price of the *i*th component falls relative to the overall price, P87, the weight of the *i*th component increases in the calculation of the growth of real GDP. Thus, in situations where supply-induced declines in relative prices, such as with computer and other high-tech products today, raise the relative growth of the associated real components, the weight of the fastest growing components increase with time. This is the crux of the substitution bias problem. As we shall see below, chain-type GDP avoids this difficulty because the growth of real GDP turns out to be the weighted average of the growth of the physical quantities of its components, where the weights are shares of the *i*th nominal component to nominal GDP, which is, of course, invariant to the base period.

II.2) Chain-Type GDP: The Basic Approach

The fundamental difference between fixed-weight GDP and chain-type GDP is that fixed-weight GDP values product at the prices of some fixed period, while chain-type GDP values product at "current" prices. Obviously, one can't calculate directly the level of real GDP while valuing product at current prices, since this is just nominal GDP, which we already observe directly. One can, however, calculate the *change* in real GDP valued at current prices. Thus, chain-type GDP is fundamentally a concept about the *change* in real GDP.

In the discussion that follows, $G_t = X_t/X_{t-1}$ is referred to as the growth in X. Typically, for macroeconomic time series, G will have a value in the neighborhood of 1, something slightly greater than 1 if you're lucky, and something less than 1 if you're not. This leads directly to the concept of "chaining." If you can't compute the level of real GDP directly, but you can compute the growth in GDP, then the current level of the index can always be computed by the construction of a *chain* of growth terms from an initial index value.

(7) $$Y_t = Y_0 G_1 G_2 G_3 \ldots\ldots G_t$$

If, in the base year, Y_0 is set equal to 100, then the units of Y_t are that of a "quantity index." If, instead, Y_t is set equal to the value of nominal GDP in the base year, then Y_t is denominated in dollars. This latter approach is taken in the US, where the base year is 1992. Note that chaining can work in both directions in time.

The computation of G can be accomplished using any of several weighting schemes. Consider first a one-period Laspeyres *chain-type* index comparing the value of this period's quantities to the value of last period's quantities valued at last period's prices:

(8) $$G_{L,t} = (p_{C,t-1}Q_{C,t} + p_{I,t-1}Q_{I,t}) \,/\, (p_{C,t-1}Q_{C,t-1} + p_{I,t-1}Q_{I,t-1})$$

or alternatively, in terms of price indices and dollar-denominated real quantities, (8’) $G_{L,t} = (P_{C,t-1}C_t + P_{I,t-1}I_t) / (P_{C,t-1}C_{t-1} + P_{I,t-1}I_{t-1})$

For purposes of making the exposition general, we have dropped the “87” designation from C87 and I87, as can be seen in expression (8’). We prefer expression (8’) because it employs the price indices and dollar-denominated real quantities we are accustomed to working with. Note that fixed-weight GDP is a Laspeyres index, but with price weights from a fixed period.

After a little manipulation, (8’) can be re-written as a weighted average of the growth of C and I:

(9) $G_{L,t} = \beta\$_{t-1}(C_t / C_{t-1}) + (1 - \beta\$_{t-1})(I_t / I_{t-1})$,

where the weight, $\beta\$_{t-1}$, is the *lagged* share of nominal consumption in nominal GDP, or $\beta\$_{t-1} = C\$_{t-1}/Y\$_{t-1}$. The key to getting weights which are nominal shares rather than real shares is that the prices in the numerator of expression (8) are from the same period as the prices and quantities in the denominator of (8), i.e. they are “current” prices. If instead, the prices in (8) were from some fixed (base) year, as in (5), the weights would turn out to be real shares.

Next, consider a one-period “Paasche index” comparing the value of this period’s quantities to the value of last period’s quantities valued at this period’s prices:

(10) $G_{P,t} = (p_{C,t}Q_{C,t} + p_{I,t}Q_{I,t}) / (p_{C,t}Q_{C,t-1} + p_{I,t}Q_{I,t-1})$,

or alternatively,

(10’) $G_{P,t} = (P_{C,t}C_t + P_{I,t}I_t) / (P_{C,t}C_{t-1} + P_{I,t}I_{t-1})$.

With a little manipulation, this too can be re-written as a weighted average of the growth in C and I:

(11) $$G_{P,t} = [\beta\$_t(C_t / C_{t-1})^{-1} + (1 - \beta\$_t)(I_t / I_{t-1})^{-1}]^{-1}$$

Here, the weight $\beta\$_t$ is the *current* share of nominal consumption in nominal GDP. Note again, that since the changes in the real quantities are independent of the base year, and since the nominal component shares are independent of the base year, the growth term itself is independent of the base year.

It is a property of the Laspeyres index that it overstates the value of quantity changes while a Paasche index understates them. A better measure is a Fisher Ideal index which is the geometric average of the Laspeyres and Paasche indices:

(12) $$G_{F,t} = [\, G_{L,t}\, G_{P,t}\,]^{.5}$$

A corresponding measure of the Fisher Ideal Index for the aggregate price level (P_t) can be constructed in an analogous manner by reversing the roles of the prices and quantities in expressions (8') and (10') and applying expression (12) to the resulting Laspeyres and Paasche indices. As we have just seen, expressions (8') and (9) are interchangeable ways to calculate the Lapeyres index and expressions (10') and (11) are interchangeable ways to calculate the Paasche index, and we can thus describe the weighting process either in terms of price weights or shares of nominal GDP. However, while it is sometimes more intuitive to describe the difference between the Laspeyres and Paasche indices in terms of the nominal-GDP share weights, as we shall see below, it is computationally cleaner and offers more flexibility to use the price-weight formulations. Once the Fisher Ideal growth term (12) is calculated, expression (7) can be used to calculate the corresponding level of real chain-type GDP or chain-type GDP index.

A nice feature of the Fisher Ideal measures is that if real GDP is denominated in dollars, then nominal GDP is still the product of real output and the price level:

(13) $$Y\$_t = P_t\, Y_t\, .$$

Comparing expression (13) to expression (4) clarifies that, at least in concept, the chain-type methodology for aggregation has nothing to do with the measurement of nominal GDP, only its decomposition into real output and price. The formulae also make clear, however, that as a general rule chain-type GDP is not the arithmetic sum of its components which is problem I.1 listed in section I.

II.3) Implementation Details

What we have just outlined is the *conceptual approach* used by BEA to construct its chain-type measures of real GDP and the aggregate price level, but the actual implementation of this approach is complicated enough to warrant some additional discussion. We begin by considering the annual data which, with the exception discussed in the following paragraph, are constructed as described above with the time period "t" defined as a calendar year; note that, in this process, the underlying data on prices and quantities are averaged to an annual basis before aggregating them.

The aforementioned exception occurs in the ultimate year of the historical time series because BEA has decided not to include a completed calendar year into the annual weighting scheme until after the first annual revision for the year in question. This usually occurs in July of the following year. Therefore, in the latest available full year of data, GDP is not always calculated by growing the previous year's level with the Fisher index for the latest year; doing so implicitly would utilize the Paasche index based on the latest year's preliminary estimates of prices, or, alternatively, preliminary estimates of shares in nominal GDP. Instead, it is calculated using only the Laspeyres index for the latest year, which, the reader will recall, is based on the previous year's prices. Hence, at the end of the otherwise "Fisher Ideal" time-series for GDP is appended a "Laspeyres tail." With the annual benchmark revisions, usually in July, data for the latest full year will be revised and re-weighted to become a true Fisher Ideal index. This is the annual version of problem I.3, the use of two different weighting schemes.

Things get even more complicated when we consider the quarterly data because they are not true Fisher Ideal indices, even before the start of the Laspeyres tail. They would be true Fisher Ideal indices if they were constructed as described above with "t" defined as a calendar quarter. But in fact, to reduce volatility in the quarterly series, BEA decided to weight quarterly changes in quantities (and prices) by prices (and quantities) that are <u>annual averages from two adjoining years, rather than the current and lagged quarterly weights</u>. BEA felt that the quarter-to-quarter volatility in the quantities and prices that serve as the weights in the aggregation formulae would impart *unacceptable* volatility to the aggregated components. It is not clear what exactly was meant by *unacceptable*, or what, if any, experimentation or other evidence led them to this conclusion.

Figure 1

Weighting Quarterly Quantity Changes:

Fisher Ideal History

<table>
<tr><td colspan="4">Annual Prices for Laspeyres Index</td><td colspan="4">Annual Prices for Paasche Index</td></tr>
<tr><td colspan="4">1994</td><td colspan="4">1995</td></tr>
<tr><td>Q1</td><td>Q2</td><td>Q3</td><td>Q4</td><td>Q1</td><td>Q2</td><td>Q3</td><td>Q4</td></tr>
<tr><td></td><td></td><td colspan="4">Quarterly Fisher Index</td><td></td><td></td></tr>
</table>

To understand how this weighting scheme works, consider the years 1994 and 1995. The quarterly Laspeyres growth terms used to chain to the level of real GDP from 1994:Q3 through 1995:Q2 all are calculated using price weights computed as annual averages over calendar year 1994, while the corresponding Paasche indices use 1995's prices. This is shown in Figure 1. The use of 1995's prices is then extended into the Laspeyres tail of the quarterly data, which currently begins in the third quarter of 1995. This is the quarterly version of problem I.3. (See Figure 2.) Note that in the Laspeyres tail, recent quarterly data are once again constructed as fixed-weight indices, since the annual prices from the so-called "anchor year" (1995 in this discussion) are used as the weights in the aggregation of the real components.

Figure 2

Weighting Quarterly Quantity Changes:

The Laspeyres Tail

<table>
<tr><td colspan="4">Annual Shares for Laspeyres Tail</td><td colspan="4"></td></tr>
<tr><td colspan="4">1995</td><td colspan="4">1996</td></tr>
<tr><td>Q1</td><td>Q2</td><td>Q3</td><td>Q4</td><td>Q1</td><td>Q2</td><td>Q3</td><td>Q4</td></tr>
<tr><td colspan="2"></td><td colspan="6">Quarterly Laspeyres Index</td></tr>
</table>

One question that immediately springs to mind is this: how could the annual average of quarterly data constructed this way possibly equal the annual data constructed as described above? The answer is, it shouldn't. However, BEA makes an *ex post* adjustment to the quarterly data so that the average of the four quarterly values within a calendar year does in fact just equal the reported annual figure. Yet, more than one year after the conversion to chain-type GDP as the featured measure, BEA has still not formally and publicly documented how this "interpolation," as they call it, is accomplished. This is problem I.6. There is not much more to say about this problem. Historically, the published quarterly data do average to their annual counterparts. In simulation, we use annual averaging of the quarterly data, component by component. Until we can learn more about BEA's interpolation procedures, this is about all we can do.

As should be clear from the above discussion, the quarterly data are not true Fisher Ideal indices. As a result, they do not have the property that the nominal magnitude is the product of the real magnitude and the chain-type price index. This is problem I.4, and it applies only to the quarterly aggregates. However, one can still compute an implicit deflator for any aggregate as the nominal magnitude divided by the real. These implicit deflators are identical to the chain-type indices for annual data, and close, but not exactly equal to the chain-type indices for quarterly data. Solutions to this problem will be discussed further in section V.

Another implementation detail that is a problem from a modeling perspective, is that every published aggregate time series is constructed from the lowest level of detail available. In the US national accounts there are roughly 1300 "elemental" components of GDP. Since even the largest structural macro models of the US attempt to model far far fewer of the elemental real and price components, modelers are stuck with constructing aggregates from aggregates. There is no guarantee the resulting aggregation will exactly match the official published figures. This is problem I.5. A solution to this problem in simulation is discussed in section V.

The use of different weighting schemes for deep history and recent history, and especially the use of a fixed-weight Laspeyres index in the Laspeyres tail poses an interesting problem. Since the Laspeyres tail is extended forward in time with the first release of each additional quarter of data, there arises the possibility of substitution bias creeping back into the recent historical and forecasted data. This may not be a serious problem for a forecast that extends just a few quarters into the future, but certainly is for a forecast with a time horizon measured in years. Therefore, we have structured the aggregating identities in the Macroeconomic Adviser's model such that users have the option of generating quarterly forecasts of aggregates that are constructed as a fixed-weight Laspeyres index, or as a true quarterly Fisher index, or as an annual Fisher index, that is, extending the use of annual weights. These details are covered in section V.

In the following three sections we address the three major tasks that modelers must complete in order to properly convert a fixed-weight model to a chain-type model. This discussion draws on our own experience in re-structuring the Washington University Macro Model (WUMM) to accommodate chain-type GDP. This model was built by the principals of Macroeconomic Advisers, is maintained by MA, and is made available to its clients under license. WUMM is one of the class of "large" quarterly structural macro models that traces its origins back to the MIT-Penn-SSRC modeling effort. We offer below our particular set of solutions to the problems indicated above. There certainly may be other reasonable solutions.

III) Model Re-estimation with Chain-Type GDP

Least squares estimators are both biased and inconsistent if the independent variables in a regression are mismeasured. Hence, if chain-type data are "true", then estimation with them should produce parameter estimates that more accurately reflect underlying utility and production functions than do the biased and inconsistent estimates derived from fixed-weight constructs. This doesn't mean, however, that regressions will "fit" better with chain-type data. That depends upon the variances and covariances of both the dependent and independent variables in one's regressions, old and new. Generalizations are not really possible beforehand. We do suspect, however, that the use of annual weights in the computation of the quarterly data and the *ex post* adjustment made to the quarterly figures in order to achieve consistency with the annual numbers have implications for serial correlation in the residuals of our equations. Since these "interpolations" remain undocumented, we remain unsure of their effects.

When running regressions with chain-type data, one should think carefully about the issue of scaling, although this has turned out to be less of a concern than it was prior to the actual switch to chain-type GDP. In the months leading up to the switchover to chain-type GDP, BEA had considered featuring the pure index form of GDP and its components. That is, both GDP and the price index for GDP, as well as every other component and price index would have a value of 1 (or 100) in the base year. BEA eventually elected to feature the 1992-dollar denominated indices. In the days before chain-type GDP, we were regressing consumption on household income and wealth with everything measured in 1987 prices. The estimated coefficients had familiar and straightforward interpretations as marginal propensities to consume. We take a certain comfort from these numbers because we can place them in the context of a long literature on the consumption function and so have some immediate sense of what constitutes an acceptable empirical and theoretical result.

Table 1

ESTIMATES OF SERVICE CONSUMPTION

Dep Var: Service Consumption (Per Capita)

Sample: 1966:Q4 - 1991:Q4

	(1) 1987 Prices	**(2) Chain Index**	**(3) Chain Dollars**
Rebate Income	0.08141	0.00470	0.07689
	(1.43877)	(1.33123)	(1.33123)
Labor Income	0.26483	0.01603	0.26240
	(7.31120)	(7.41878)	(7.41878)
Medical Transfers	0.99825	0.05653	0.92088
	(3.84184)	(3.65765)	(3.65765)
Other Transfers	0.46669	0.02747	0.45020
	(3.58657)	(3.69483)	(3.69483)
Equity Values	0.02361	0.00173	0.02833
	(2.72113)	(3.51620)	(3.51620)
Other Net Worth	0.05001	0.00311	0.05089
	(7.11594)	(7.77426)	(7.77426)
Adj R-Squared	0.99944	0.99945	0.99945
Durbin-Watson	1.78539	1.84071	1.84071
Rho	0.84515	0.81802	0.818
% R.M.S.E.	0.68510	0.63960	0.63960

If the series on consumption and income we use in our regressions were measured as index numbers rather than in dollars, the estimated coefficients would not be subject to familiar interpretations until we scaled the results back into an intelligible form. True, not all regressions are subject to this nuisance. For example, many of the relationships in our model are log-linear with coefficients interpreted as elasticities that are independent of how the data are scaled. Still others are in terms of nominal quantities and so are unaffected by the new methodology. But in cases where ease of interpretation is an issue, we decided long ago to use the real quantity indices denominated in 1992 dollars before performing any empirical work. As it turned out, BEA is featuring the 1992-dollar denominated data, and so it is readily available.

Initially we worried that re-estimation of WUMM using the new data could so alter some of its key relationships that the fundamental properties -- perhaps even the stability -- of the model might be affected. In fact, we've found that most equations look pretty much the same whether estimated on the old or new data. As a case in point, consider one of the largest components of GDP in WUMM: consumer spending on services. It is modeled as a function of tax rebates and surcharges, disposable labor income, medical transfer income (mostly Medicare and Medicaid benefits), other kinds of transfer income (mostly Social Security benefits), the equity portion of household net worth, and other net worth. Our old regression is summarized (we've spared the gory details of the distributed lags) in Column 1 of Table 1.

Everything is measured in 1987 prices and per capita. T-statistics are shown in parentheses, and some regression summary statistics are also shown at the bottom of the table. The coefficients are subject to the usual interpretation; i.e., the MPC out of labor income is 26 cents on the dollar, the MPC out of equity values is 2.4 cents, and so forth.

Next, we re-ran the regression using the quantity index for spending on consumer services (per capita) as the dependent variable and computing real incomes and wealth through division by the alternative price index for total PCE rather than its implicit deflator. These results are shown in column 2 but, in this form, it is hard to make sense of the coefficients -- the scaling problem. Finally, in column 3 is shown the same regression as in column 2 but with the quantity index for PCE-services scaled to 1987 dollars. Now columns 1 and 3 are more comparable and, as you can see, the qualitative characteristics of the regression haven't changed too much.

Which equation is "better?" With the chain-type data, t-scores generally are somewhat higher. In the case of the estimated MPCs out of net worth, the differences are large enough to get our attention, especially since the link between equities and the consumption of services -- a very important relationship in the model -- is about 20% larger when estimated with the chain-type data. The adjusted R^2 seems to give the slightest of edges to the "old" equation, but that comparison is misleading because the regressions have different dependent variables to which are applied slightly different corrections for first-order serial correlation. To circumvent that obstacle, we computed the percent root-mean-square error of each equation without the advantage of the rho-correction, which is how equations are coded in our model. For the old equation, the RMSE is 0.6851%; for the new, 0.6396% -- a 6.6% improvement. On balance, we'd say, "usher out the old and bring in the new!"

In other equations where relative prices play a more direct role the parameter estimates can and do change more, but the bottom line is this: you can proceed pretty much as usual with your regression work. We recommend that if given the choice of using pure index numbers or dollar-denominated values, use the dollar-denominated values. It does no harm, and can assist in the interpretation of some parameters. The fit of some familiar specifications will probably get better, some might get worse, and some coefficients will change by amounts that could be worth worrying about; but don't expect to unearth some startling new macro relationship that has gone unnoticed over the years.

IV) Simulation Should be Independent of the Aggregation Scheme

If it is true that the economy evolves through time independent of our measurement of it, then it should also be true that model simulations should be independent of the aggregation scheme used. This becomes an issue because of the use of multiple aggregation schemes. If there were only one weighting option, e.g., all historical and forecast data were constructed as true quarterly Fisher Ideal indices, then the aggregating identities in the model would be coded for this method of aggregation and we would be done. However, in the US, the construction of recent historical data as fixed-weight Laspeyres indices, in the Laspeyres tail, requires that we offer an option of simulating the model so that it produces aggregate measures constructed this same way. Users whose forecast horizon is one, two, or three quarters, and there are many, want to forecast the data that BEA will be releasing. On the other hand, long-term projections that produce fixed-weight Laspeyres aggregates are subject to substitution bias creeping back into the simulation. Therefore, a true chain-type aggregation method must also be available for these longer projections.

We have chosen to offer three types of aggregation schemes in simulation: annual Fisher, Laspeyres Tail, and quarterly Fisher. Each of these can be selected interactively by a user simulating the model. Table 2 summarizes the alternative weighting schemes used historically and available in simulation. Section V covers the details of how the three alternative aggregation schemes are handled in simulation.

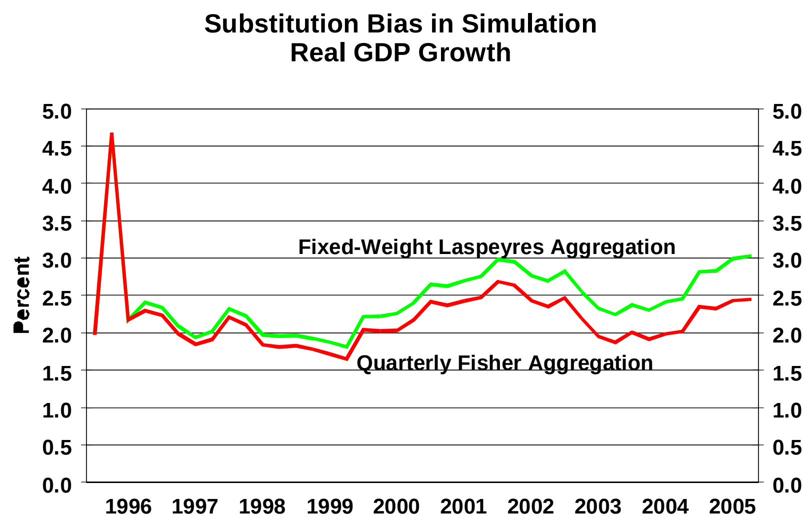
Table 2
Aggregation Schemes in Chain-type GDP

When used:	Type of Aggregation	Weights
Historical Data		
"Quarterly Deep History" Quarterly data where Paasche weights are available	Fisher	Quarters 3,4,1,2 - Annual Weights Laspeyres from year including qtrs 3 & 4 Paasche from year including qtrs 1 & 2
"Quarterly Laspeyres Tail" Quarterly data where Paasche weights are not available	Laspeyres	Laspeyres Annual Weight from the anchor year - currently 1995
"Annual Deep History"	Fisher	Laspeyres Annual Weight from Prior Year Paasche Annual Weight from Current Year
"Annual Recent History"	Laspeyres	Laspeyres Annual Weight from the anchor year - currently 1995
Forecast Data		
Quarterly Short-term Forecast	Laspeyres	Laspeyres Annual Weight from the anchor year - currently 1995
Quarterly Long-term Forecast	Fisher	Laspeyres Quarterly Weight from Prior Qtr Paasche Quarterly Weight from Current Qtr
Quarterly Long-term Forecast	Fisher	Laspeyres Annual Weight from "Prior" Year Paasche Annual Weight from "Current" Year

Lest you think we have gone to a lot of trouble for no good reason, we provide in figure 3 a comparison of the simulated growth of GDP constructed as a fixed-weight Laspeyres index that is the extension of the Laspeyres tail, and the growth of GDP constructed as a quarterly Fisher Ideal index. One of the primary reasons for the difference in the two reported growth paths is the assumed 12% annual rate decline in the price of the computer component of

Producers' Durable Equipment (PDE) relative to the price of other PDE. Even with this quite modest relative price decline (the actual has been running at 15% - 20%) for computer equipment, the difference in GDP growth quickly reaches a couple of tenths of a percentage point and exceeds 1/2 percentage point by the end of the ten-year simulation horizon.

Figure 3



<u>How then can one construct a model with underlying simulation properties independent of the aggregation scheme used in simulation?</u> The answer is to remove from the simultaneous block of the model, all aggregates that depend on the aggregation scheme. This can accomplished by either replacing these aggregates with their elemental components, or by replacing these aggregates with additional variables constructed such that they are always computed the same way in simulation regardless of the aggregation scheme in use. For example, in WUMM, the real value of nonfarm business output less housing, a variable we call QB, is the key "scale" variable in a number of important behavioral equations. Of course, as BEA reports this series, it is an annual Fisher Ideal index in deep history, and a fixed-weight Laspeyres index in recent history. Will it be yet a third type of index in simulation? To cut through this problem we construct a variable we call QBADJ. In WUMM, QBADJ is always a quarterly Fisher index built from the elemental components in the model. This series is then used in the estimation

and simulation of several key equations. There are eight such “auxiliary” aggregates in WUMM.

This approach reduces the problem of multiple aggregation schemes to basically a table-writing problem. All official aggregates in the model are calculated post simultaneously, according to the aggregation scheme selected by the user. Since none of the “official” aggregates appear in the simultaneous block of the model, switching aggregation schemes has no impact on any of the simulation values of any variables, other than the official aggregates themselves, or other series which are recursive.

V) Restructuring the Identities in the Model

Another major task facing a macro modeler in the switch to chain-type GDP is the restructuring of a large number of identities in the model. This task is necessitated by problem I.1, the nonadditivity of real GDP, or, more generally, the fact that the method of aggregating real quantities and prices has changed. In addition, the fact that for quarterly data real times price no longer yields exactly the nominal magnitude, problem I.4, requires restructuring each of these identities. The fact that BEA uses different weighting schemes for deep history and recent history, with newly-released aggregate data constructed as fixed-weight Laspeyres indices, problem I.3, requires that the new identities be specified to allow for multiple weighting schemes. Otherwise, users performing long-term projections would again face the problem of substitution bias creeping back into their forecasts.

The two types of identities to be restructured are:

1) All aggregation identities for real components and price indices of GDP. We shall refer to these as *aggregation identities*.

2) All identities that construct nominal magnitudes from real quantities and price indices. We shall refer to these as *PXQ identities*.

V.1) Aggregation Identities in Model Simulation

As we indicated earlier, thinking of GDP growth as a weighted average of the growth of its components, where the weights are nominal GDP shares, is quite intuitive. However, the price-weight formulation turns out to computationally cleaner and more easily accommodates the kind of flexibility in simulation discussed above.

The aggregation of two or more real components of GDP can be illustrated by continuing with our two good economy. Below we repeat expressions (8'), (10'), and (12). These constitute the basic set of aggregation calculations that must be performed for each real aggregate and each price aggregate.

(8') $G_{L,t} = (P_{C,t-1}C_t + P_{I,t-1}I_t) / (P_{C,t-1}C_{t-1} + P_{I,t-1}I_{t-1})$

(10') $G_{P,t} = (P_{C,t}C_t + P_{I,t}I_t) / (P_{C,t}C_{t-1} + P_{I,t}I_{t-1})$

(12) $G_{F,t} = [\ G_{L,t}\, G_{P,t}\ ]^{.5}$

To illustrate how these expressions are coded to allow for the possibility of multiple weighting schemes, we write (8') and (9') below, substituting price weight variables for the price indices.

(8'') $G_{L,t} = (W_{C,L}C_t + W_{I,L}I_t) / (W_{C,L}C_{t-1} + W_{I,L}I_{t-1})$

(10'') $G_{P,t} = (W_{C,P}C_t + W_{I,P}I_t) / (W_{C,P}C_{t-1} + W_{I,P}I_{t-1})$.

There are two price weight variables for each real GDP component, a Laspeyres price weight and a Paasch price weight. Once the aggregating identities are coded this way, switching aggregation schemes is accomplished by re-computing the price weight terms in accordance with the aggregation scheme.

For the Laspeyres tail, the values of $W_{C,L}$ and $W_{C,P}$ are the annual average price of C in the anchor year, currently 1995. The values of $W_{I,L}$ and $W_{I,P}$ are the annual average price of I in the anchor year. You will notice that this generates values of $G_{L,t}$ and $G_{P,t}$ that are identical. And indeed, this is the desired result, since we want expression (12) to yield the fixed-weight Laspeyres index. Because the aggregates are fixed-weight indices in the Laspeyres tail, the price weights need only be computed once, and then applied iteratively over the forecast horizon.

For the quarterly Fisher index, $W_{C,L}$ and $W_{C,P}$ are the lagged and current price index for C, respectively, and $W_{I,L}$ and $W_{I,P}$ are the lagged and current price index for I, respectively. Feeding the resulting values of $G_{L,t}$ and $G_{P,t}$ into expression (12) yields a true quarterly Fisher index. In this case, the price weights must be re-computed each quarter in the simulation horizon.

For the annual Fisher index, $W_{C,L}$ and $W_{I,L}$ are the annual average price indices for C and I, respectively, in the year containing quarters 3 and 4, and $W_{C,P}$ and $W_{I,P}$ are the annual average price indices for C and I, respectively, in the year containing quarters 1 and 2. Thus, as the Fisher index is computed forward through time, the price weights are re-computed every four quarters.

Two points are in order about the discussion above. First, we are treating C and I as if they are elemental components of GDP, when in fact they are themselves aggregations of elemental components. We sometimes refer to this process as calculating "a Fisher of Fishers." In WUMM, there are roughly 40 components of GDP at the lowest level of aggregation, none of which are truly elemental in that BEA produces more disaggregated data. Second, we have carried out the entire exposition in terms of aggregating real quantities. There is a parallel and entirely analogous set of expressions for aggregating prices, where the roles of prices and quantities are reversed.

Once the growth terms are calculated, the level of the index must be computed. A condensed version of expression (7) is shown below.

(7') $$Y_t = Y_{t-1}G_t$$

This expression shows how the current level of GDP is constructed by chaining to the previous level with the growth index. However, as noted in section I, the construction of the aggregate data from the lowest level of elemental components available will mean that construction of the aggregate with anything but the lowest level of detail will not exactly replicate the official figures. Therefore, we calculate historically a residual term R_t:

$$R_t = Y_t / Y_{t-1} G_t \quad (14)$$

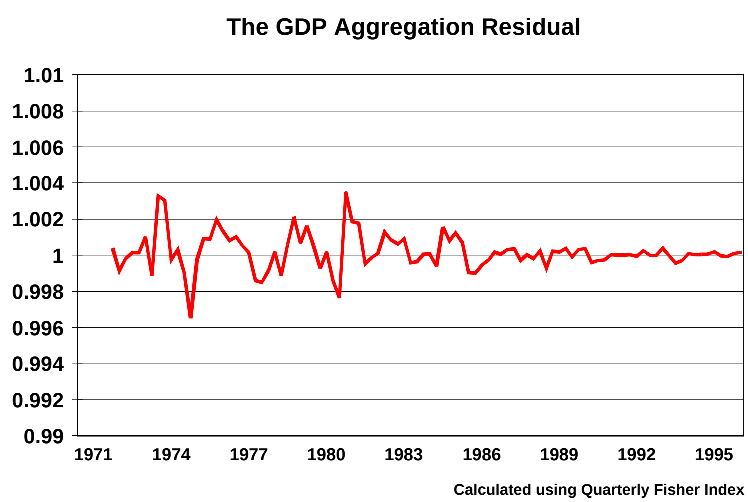
These residuals are calculated historically for every quantity and price aggregate in the model, maintained in the model database, and can be displayed with the model simulation software. Because the term G depends on the aggregation method employed, so too do the residual terms. Switching aggregation methods requires recalculating historically the residual terms. For simulation purposes we need to incorporate the residual, and (7') becomes:

$$Y_t = Y_{t-1} G_t R_t \quad (15)$$

In simulation these residuals can be altered by the user, but our experience suggests that they are best set equal to one in the forecast horizon. Values other than one imply a boost or a drag to the growth of Y relative to the underlying components. Figure 4 shows the GDP aggregation residual.

It should be made clear that this residual does not make the components of GDP additive.

Figure 4



They will still fail to add-up to GDP. Presenting the components in a table with GDP, usually involves calculating the additive residual and including it as a separate line in the table.

There has been some discussion in modeling circles in the US about the properties of these aggregation residuals, and whether anything useful could be found from an analysis of them historically. Our own view is that such an analysis would yield little of value. Because of the existence of the Laspeyres tail, any statistical analysis of the properties of these residuals in deep history, does not tell us anything about the properties of the residuals in the Laspeyres tail, or the behavior of the residuals in the forecast period. And, the five or six quarters in the Laspeyres tail are an insufficient number of observations to perform any formal tests.

If you are beginning to suspect that several of the modeling difficulties discussed here can be traced back to the decision in the US to employ two different weighting schemes over history, you are correct. A single aggregation method, such as quarterly Fisher, would have obviated the need for multiple aggregation methods, would have allowed us to learn something useful from the aggregation residuals, and as we will see below, would have allowed us to drop one whole set of residuals altogether.

V.2) PxQ Identities in Simulation

Conceptually, for true Fisher Ideal indices of real output and prices, it is the case that expression (13), repeated here, holds identically.

(13) $$Y\$_t = P_t\, Y_t\ .$$

In the US implementation of chain-type GDP (13) does hold for annual data, but does not hold for quarterly data. Therefore, both historically and in the forecast period, we calculate a "PxQ residual" of the form:

(16) $$R_t = Y\$_t / P_t\, Y_t$$

A residual of this type is computed for every time period and for every PxQ identity. These residuals are stored in the model database, can be viewed with the model simulation software, and can be manipulated by the user in the forecast period. These residuals are independent of the aggregation scheme. Incorporating the residual in the PxQ identity (13) yields:

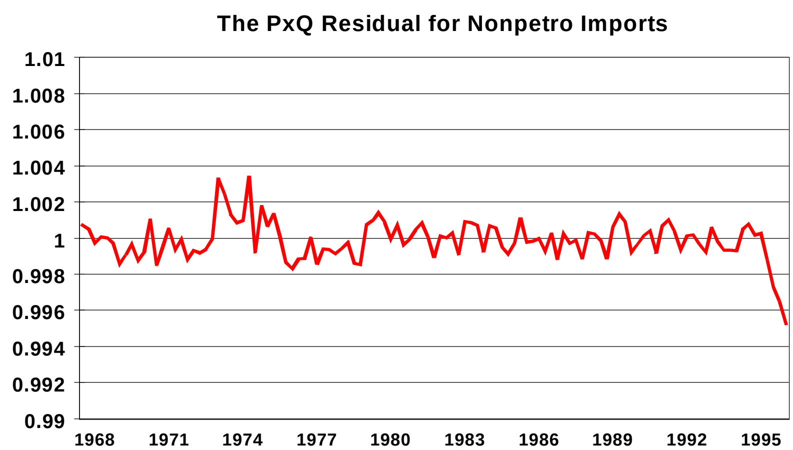
(17) $$Y\$_t = P_t \, Y_t \, R_t.$$

There may be some exploitable information in these residuals, especially if one is operating in the Laspeyres tail. As Figure 5 below indicates, there is sometimes a clearly discernible trend in the residual within the range of the Laspeyres tail.

This residual appears to be equal to 1 with approximately random errors, up until the start of the Laspeyres tail. From that point the errors are decidedly one-sided. We are at a loss to explain why this happens, and have not received an answer from BEA as to why this is occurring.

The large deviation from 1 at the end of the historical data makes it imperative that users have a way of extending this residual into the forecast period. Otherwise they would impart a large jump in the first forecast quarter to any nominal magnitude so constructed.

Figure 5



VI) Summary and Concluding Remarks

The switch to chain-type GDP eliminates the substitution bias problem that plagues the fixed-weight data we have been accustomed to working with. This alone makes it worth the facing the difficulties occasioned by the switch. Nevertheless, modelers must be prepared to take the necessary steps to re-structure their models to accommodate the new regime. The major tasks are:

1) Re-estimate the behavioral relationships. Generally, where aggregates are employed on the left or right hand side of equations, eliminating the bias inherent in the fixed-weight data will improve the regressions. However, one must pay special attention to issues of scaling, especially where coefficients have familiar interpretations. For this reason, it makes sense to use dollar-denominated quantity indices where possible.

2) If more than one aggregation scheme is to be employed, and it may well be a good idea, modelers must be prepared to re-structure their models to remove any aggregate series from the simultaneous block of the model and reduce aggregation to a table-writing problem by making it totally post simultaneous.

3) Be prepared to add possibly thousands of lines of additional code to accommodate the new aggregation method. Even then, it will probably be necessary to incorporate aggregation and PxQ residuals to insure that the model can replicate the official figures.

The year and one half we spent re-structuring, and re-estimating the model was difficult to say the least. It was made more difficult by the lack of clear documentation of the important implementation details discussed in this paper, and others that we have not included. The implementation details are important, and should be subject to the review of the user community, well in advance of the actual switch to chain-type GDP. This will give users adequate time to make the needed changes.

Finally, even today, more than one year beyond the switch to chain-type GDP in the US, there is a gross lack of appreciation for the problems related to working with chain-type GDP. To the extent that groups such as ours have handled the ugly details and incorporated into our models the needed modifications, perhaps we have made it a kind of black box. Indeed, this is the case in our system. Users can be totally ignorant of the details of chain-type GDP and still generate results that observe the rules of its construction.

It behooves the statistical agencies anticipating a switch to chain-type GDP to start well in advance educating the user community about why chain-type GDP is a superior product, and how to use it intelligently.

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